For each generation bus (except the slack bus), we have the active power generation equation and the corrisponding phase angle. We write the equation as

$$\boxed{\Delta P_K = P_K^{spec} - P_K^{calc.}}$$

note sometimes "spec" is is written as scheduled.

For each load bus, the active and reactive powers are specified, and the unknowns are $|V_K|$ and δ_K, Then we write:

$$\Delta P_K = P_K^{spec} - P_K^{calc} \underset{\text{we hope.}}{\Longrightarrow} \emptyset$$

$$\Delta Q = Q_K^{spec} - Q_K^{calc} \Longrightarrow \emptyset$$

where,

$$P_K^{calc} = \sum_{j=1}^{n} \left|\bar{V}_K \bar{Y}_{Kj} \bar{V}_j\right| \cos(\delta_K - \delta_j - \theta_{Kj})$$

$$Q_K^{calc} = \sum_{j=1}^{n} \left|\bar{V}_K \bar{Y}_{Kj} \bar{V}_j\right| \sin(\delta_K - \delta_j - \theta_{Kj})$$

If bus #1 is chosen to be the slack, then we know,

$$\boxed{\bar{V}_1 = |V_1| \angle 0^\circ \ pu}$$

and we want to determine P_1 & Q_1 which is done after convergence is achieved.

In our NR general set of equations, We have the variable $\underline{x}$ is the vector containing the bus voltage magnitudes. and phase angles

Δf term is the term containing the

P_K, Q_K mismatches

At the load bus, there are two equations which are given by:

$$\boxed{\begin{aligned} \Delta f_{Kp} &= P_{KO} - P_K \\ \Delta f_{KQ} &= Q_{KO} - Q_K \end{aligned}}$$

where, P_{KO}, Q_{KO} are the specified values and P_K, Q_K are the calculated values.

Taking the differential of equation 9(a) and 9(b) we obtain:

(21 A) $$P_{KO} - P_K = \sum_{j=1}^{n} \left[\frac{\partial P_K}{\partial |\bar{V}_j|} \Delta |\bar{V}_j| + \frac{\partial P_K}{\partial \delta_j} \Delta \delta_j \right]$$

(21 B) $$Q_{KO} - Q_K = \sum_{j=1}^{n} \left[\frac{\partial Q_K}{\partial |\bar{V}_j|} \Delta |\bar{V}_j| + \frac{\partial P_K}{\partial \delta_j} \Delta \delta_j \right]$$

At a voltage control bus, only equation 21(a) applies. Also, that in the summation, terms in $\Delta|\bar{V}_j|$ do not exist, for the controlled bus j, and the swing bus

Similiarly terms of $\Delta \delta_j$ do not exist for the swing bus.

Then for bus #1 for the swing bus, $2 \rightarrow m$ as Load buses (P,Q, buses) and $(m+1) \rightarrow n$ are the Voltage controlled buses. where we know P, & V, but we want Q, and δ. Then in compact form we write

(22) $$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta} & |\bar{V}| \frac{\partial P}{\partial |\bar{V}|} \\ \frac{\partial Q}{\partial \delta} & |V| \frac{\partial Q}{\partial |\bar{V}|} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |\bar{V}|}{|\bar{V}|} \end{bmatrix}$$

(23) inverted # 22

The above equation 22 & 23 have the advantages w.r.t. convergence. After solution of equation (23) the updated values are

(24A) $$|\bar{V}|^{(N+1)} = |\bar{V}|^{(N)} \left[1 + \frac{\Delta |V|}{|\bar{V}|} \right]$$

(24B) $$\delta^{(N+1)} = \delta^{(N)} + \Delta \delta^{(N)}$$

The terms of the jacobian for $k \neq j$ are

(25)
$$\frac{\partial P_k}{\partial \delta_j} = Q_{kj} \quad , \quad |\bar{V}_j| \frac{\partial P_k}{\partial |\bar{V}_j|} = P_{kj}$$

$$\frac{\partial Q_k}{\partial \delta_j} = -P_{kj} \quad , \quad |\bar{V}_j| \frac{\partial Q_k}{\partial |\bar{V}_j|} = Q_{kj}$$

for $k = j$ we have

(26)
$$\frac{\partial P_k}{\partial \delta_k} = Q_{kk} - Q_k \quad , \quad |\bar{V}_k| \frac{\partial P_k}{\partial |\bar{V}_k|} = P_k + P_{kk}$$

$$\frac{\partial Q_k}{\partial \delta_k} = P_k - P_{kk} \quad , \quad |\bar{V}_k| \frac{\partial Q_k}{\partial |\bar{V}_k|} = Q_k + Q_{kk}$$

where,

$$\boxed{\begin{aligned} P_{kj} &= |\bar{V}_k||\bar{Y}_{kj}||\bar{V}_j|\cos(\delta_k - \delta_j - \theta_{kj}) \\ Q_{kj} &= |\bar{V}_k||\bar{Y}_{kj}||\bar{V}_j|\sin(\delta_k - \delta_j - \theta_{kj}) \end{aligned}} \qquad (27)$$

and from equation 9 we have

$$\boxed{P_k = \sum_{j=1}^{n} P_{kj} \quad , \quad Q_k = \sum_{j=1}^{n} Q_{kj}} \qquad (28)$$

Other row column arguements may be used, for example, alternative P, Q rows.

Matrix inversion in general is avoided as it can be slow and complex. It is only justified if the same matrix is used several times as in some approximate variations of the method.

The figure shown below gives the flow chart for the NR method